NOTE ON THE COMBINING OF TWO PROBABILITIES BY MEANS OF A SCATTER DIAGRAM

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The final product of a forecasting technique using scatter diagrams is often in the form of a chart combining two probabilities. When this is the case, sufficient data are usually not available to define fully the positions of the individual probability lines. To aid in the placing of these lines, certain factors gained from experience and supported by intuitive reasoning can be used.

A common error encountered in the analysis of the combined probability chart is that which allows the individual probability lines to intersect the sides of the chart (see, for example, charts given in fig. 20 of [1]).

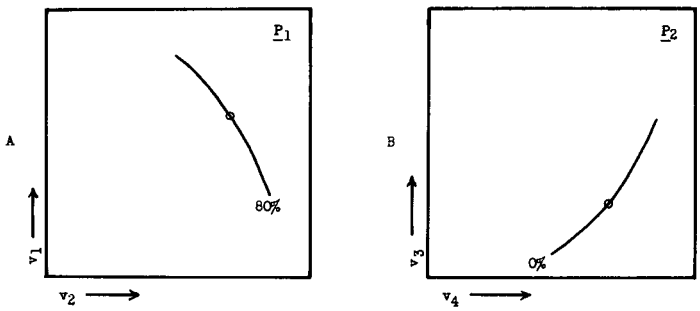

FIGURE 1.—Charts from which probabilities P_1 and P_2 are obtained. Case under consideration occurs at the 80-percent level on chart A and at the zero-percent level on chart B. Since it is a "no-rain" case, it is indicated by an open circle.

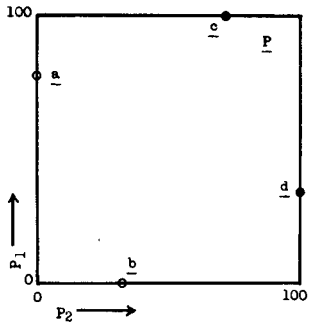

FIGURE 2.—Chart showing locations of "no-rain" cases when either probability is zero or 100 percent. "No-rain" cases are indicated by open circles and "rain" cases by solid dots. The case considered in figure 1 falls at point *a*. When a case approaches either the upper left or lower right corner of the chart, the combined probability becomes indeterminant.

It can be seen from a rather simple analysis of the data that this cannot occur. For illustrative purposes, let us consider two charts (A and B in fig. 1) which result from the combination of two pairs of variables v_1 and v_2, and v_3 and v_4, the analyses of which are expressed as probabilities of rain P_1 and P_2. Let us further consider an individual case of the developmental sample which appears in the area defined by the 80-percent line in chart A (P_1=80%) and the zero-percent area in chart B (P_2=0%), as indicated in figure 1. Since the case under consideration occurs at the zero-percent level on one of the charts it must be a "no-rain" case. When this case is entered on the final probability chart for analysis it will appear at point *a* as an open circle indicating a "no-rain" case as shown in figure 2.

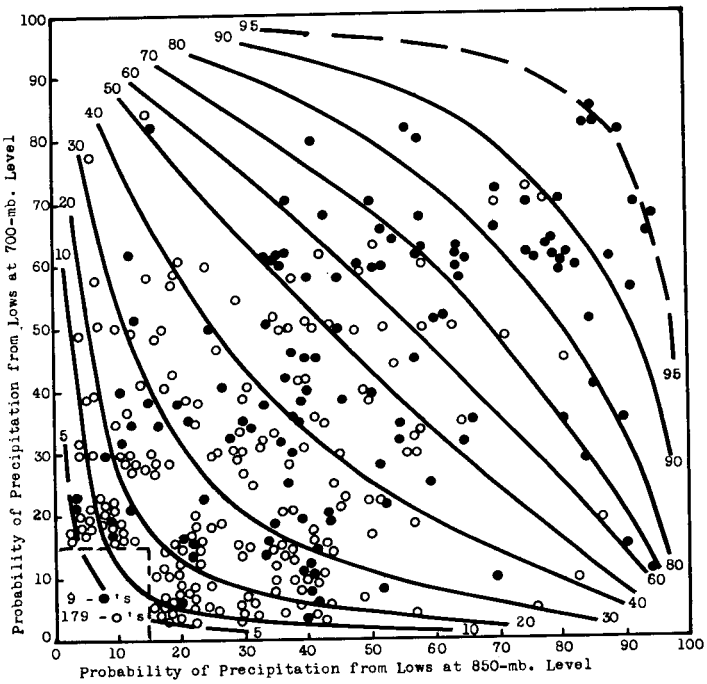

FIGURE 3.—Chart combining the probabilities of precipitation obtained from the 850- and 700-mb. levels at Salt Lake City, Utah. Analysis shows equal probability lines.

Similar reasoning will indicate that all cases falling along the left side of the chart (and also along the bottom of the chart as at point b) will be "no-rain" cases, and that cases falling on the lines $P_1=100$ and $P_2=100$ must be "rain" cases, e.g., points c and d. Therefore, it can be seen that the zero-percent probability area defined by the data must include all $P_1=0$ and $P_2=0$ values since only "no-rain" cases can appear on these two lines, and the 100-percent probability area must include the $P_1=100$ and $P_2=100$ values since only "rain" cases can appear on these lines. (When a probability is combined with a variable, it also follows that the analyzed lines should not be allowed to intersect the $P=0$ and $P=100$ values on the chart.)

The data within the chart will then establish the placing and shape of the individual probability lines between the 0- and 100-percent values. These lines should be more or less symmetrical about the line running from the lower left to the upper right corners of the chart.

From the above considerations, the typical chart giving the combined probabilities should show the low probability lines (e.g., the 10-percent line) asymptotic to the left and bottom sides of the chart, and the high probability lines asymptotic to the upper and right sides. An example of this type of analysis carried out for actual data involving the combination of probabilities of precipitation occurrence from Lows at the 700- and 850-mb. levels at Salt Lake City [2] is shown in figure 3.

REFERENCES

1. H. A. Panofsky and G. W. Brier, *Some Applications of Statistics to Meteorology*, The Pennsylvania State University, University Park, 1963, 224 pp.
2. A. F. Korte, D. L. Jorgensen, and W. H. Klein, "Probability of Station Precipitation in the Western Plateau States From 850-Mb. Lows During Winter," Techniques Development Laboratory, Weather Bureau, ESSA, Apr. 1968, 18 pp. (unpublished).

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